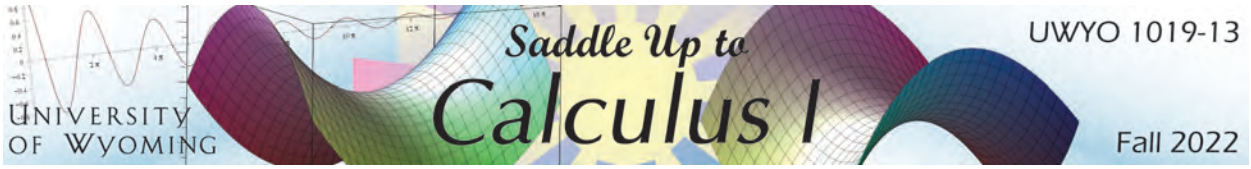
Name .....



Quiz 2

Thursday, August 18, 2022

A student is asked to find all points (x, y) satisfying $y = 1 - x$ and $y^2 = 1 - x^2$. He writes

$$\begin{aligned}
y^2 &= 1 - x^2 \\
(1-x)^2 &= 1 - x^2 \\
1 - 2x + x^2 &= 1 - x^2 \\
-2x + x^2 &= -x^2 \\
(-2+x)x &= -x^2 \\
-2 + x &= -x \qquad \text{No!} \\
2x &= 2 \\
x &= 1 \\
y &= 0 \\
(x, y) &= (1, 0) \text{ is the only solution}
\end{aligned}$$

What is wrong with the student's argument?

The error is in the middle where the student cancels x from both sides. In reality, there are two cases: either $x \neq 0$ (leading to the student's solution) or $x = 0$ (leading to the second solution $(x, y) = (0, 1)$).

The two solutions are the points of intersection of the line and circle, as shown: