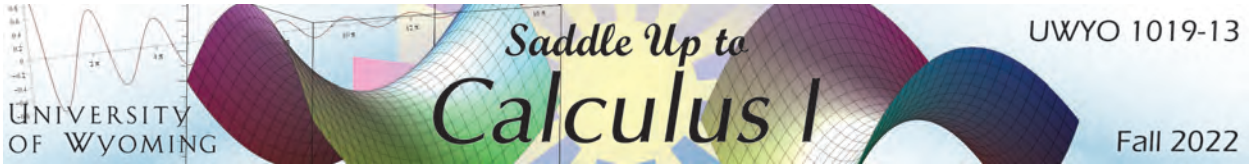

HW2

Due 11:59 pm, Wednesday, August 17, 2022 on WyoCourses

Refer to the syllabus for general expectations regarding homework (and references therein to the FAQ page for further details). In particular please read questions carefully to be sure you are answering the question that is asked. Write clearly, using correct language and notation. *Simplify your answers.* Total value of questions: 20 points.

Consider the set of positive integers $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$. A function $f : \mathbb{N} \to \mathbb{N}$ is defined by

$$f(n) = \begin{cases} 3n+1, & \text{if } n \text{ is odd;} \\ n/2, & \text{if } n \text{ is even.} \end{cases}$$

For example, $f(10) = 5$ and $f(11) = 34$. The *k-th iterate of f* is the function f^k obtained by applying f repeatedly, k times. For example,

$$f(10) = 5; \quad f^6(10) = f(f(f(f(f(f(10)))))) = 1; \quad f^7(10) = 4; \quad f^8(10) = 2; \quad f^9(10) = 1.$$

1. *(5 points)* Evaluate $f^{13}(21)$. We find $f^{13}(21) = 1$.

The first few iterates of f are

$21, 64, 32, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, \ldots$

2. *(5 points)* What is the *smallest* k such that $f^k(21) = 1$?

The smallest such k is 7, as seen in the list of iterates in #1.

3. *(10 points)* What is the *smallest* k such that $f^k(27) = 1$?

$k = 111$. (I'm not listing all the iterates this time.)