University of Wyoming · Math 2850—Fall 2024 · Department of Mathematics

Putnam Team Seminar

Practice Problems 2

Monday, September 9, 2024

1. Alan and Barbara play a game in which they take turns filling entries of an initially empty 2008×2008 array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?

2. Let f and g be (real-valued) functions defined on an open interval containing 0, with g nonzero and continuous at 0. If fg and f/g are differentiable at 0, must f be differentiable at 0?

3. Find the volume of the region of points (x, y, z) such that
$$(x^2+y^2+z^2+8)^2 \leqslant 36(x^2+y^2).$$

4. Evaluate
$$\sqrt[8]{2207-\frac{1}{2207-\frac{1}{2207-\cdots}}}\,.$$
Express your answer in the form $\frac{a+b\sqrt{c}}{d}$ where a, b, c, d are integers.

5. Prove
$$\frac{22}{7}-\pi = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx\,.$$

6. Determine all polynomials $P(x)$ such that $P(x^2+1) = P(x)^2+1$ and $P(0) = 0$.

7. Let $f_0(x) = e^x$ and $f_{n+1}(x) = x f_n'(x)$ for $n = 0, 1, 2, \ldots$. Show that $\sum_{n=0}^{\infty} \frac{f_n(1)}{n!} = e^e$.