Math 2200-01 (Calculus I) Spring 2020

Book 1

Jan 27

Calculus I: Single-variable calculus $y = f(x)$ for example (one input variable x, one output variable). Derivatives (rates of change): differential calculus.

Calculus II: also single-variable. Integral calculus.

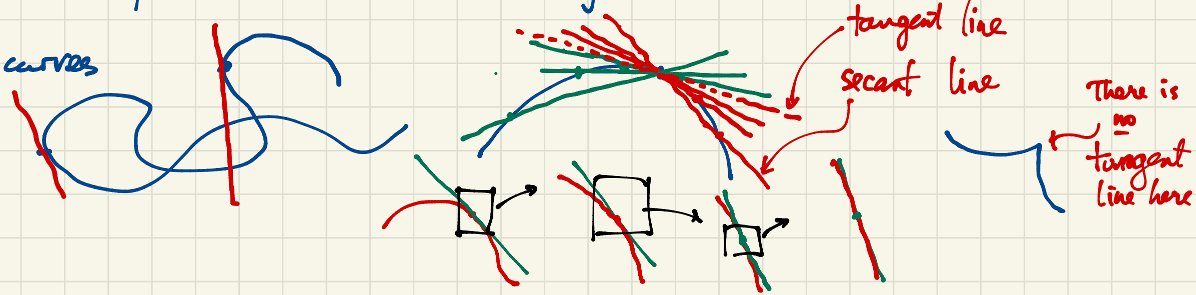
Calculus III: multivariable ie. several input variables and/or several output variables
eg. position $(x(t), y(t), z(t))$ of an object at time t: one input t, three output variables $x(t), y(t), z(t)$.

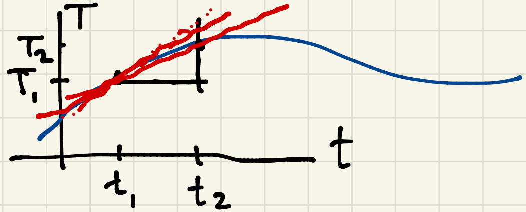
Eg. Temperature in this room as a function of position $T(x,y,z)$
(three inputs x,y,z; one output T)

Eg. Wind velocity as a function of position: three inputs x,y,z; three outputs are the components of wind velocity.

Jan 28

Tangent lines to curves





Temperature T as a function of time t

During the time interval $[t_1, t_2]$ i.e. $t_1 \le t \le t_2$

the temperature rises from T_1 to T_2.

The average rate of change of temperature during this time interval is

$$\frac{\Delta T}{\Delta t} = \frac{T_2 - T_1}{t_2 - t_1}$$

$T_2 - T_1$ ← change in temperature

$t_2 - t_1$ ← time elapsed.

= slope of the secant line from (t_1, T_1) to (t_2, T_2) on the graph.

We want to understand the instantaneous rate of change of temperature at time t_1. To determine this, first consider the average rate of change over smaller and smaller time intervals $[t_1, t_2]$ where we take $t_2 \to t_1$ (t_2 gets closer and closer to t_1).

Jan 29

Eg. In my example, $t_1 = 3$.

t_2	$\frac{T_2 - T_1}{t_2 - t_1}$	
4	2	degrees/hour
3.2	2.17	
3.1	2.19	
3.001	2.197	
2.9	2.204	
2.7	2.23	
2	2.31	

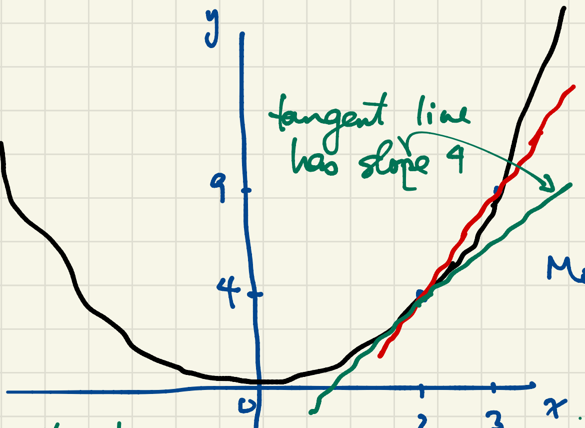
The limit is 2.2.

(The temperature at 3pm is changing at a rate of 2.2 degrees per hour.

We write $\lim_{t_2 \to 3} \frac{T_2 - T_1}{t_2 - t_1} = 2.2$

(the limit of $\frac{T_2 - T_1}{t_2 - t_1}$ is 2.2 as t_2 approaches 3).

A second example using a polynomial function $y = f(x) = x^2$. Find the rate of change of y with respect to x at $x = 2$.



The secant line joining the points $(2,4)$ and $(3,9)$ on the curve has slope

$$\frac{\Delta y}{\Delta x} = \frac{9-4}{3-2} = 5.$$

The tangent line at $(2,4)$ is $y - 4 = 4(x-2)$ i.e. $y = 4x - 4$.

More generally, if we join the points $(2,4)$ and $(x, f(x))$ on the curve, the secant line has slope

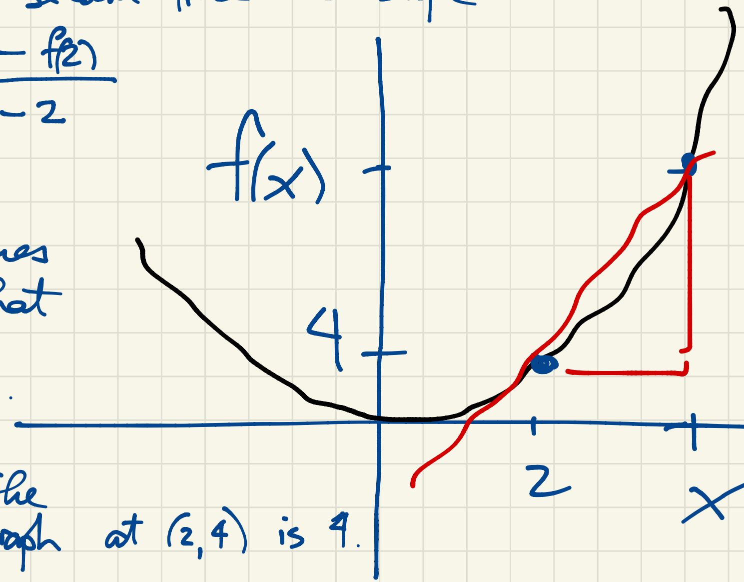
$f(2) = 4$

$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(2)}{x - 2}$$

x	$\frac{f(x) - f(2)}{x-2}$
3	5
2.5	4.5
2.1	4.1
2.01	4.01
1.99	3.99
1.9	3.9
1.5	3.5
1	3

Based on the table of values we guess that

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 4.$$



i.e. the slope of the tangent line to the graph at $(2,4)$ is 4.

If a function has a sufficiently nice formula eg. polynomial, then we have algebraic rules that provide definite ways to evaluate limits, eliminating guesswork based on the graph or table of values.

<u>Eg.</u> Find the slope of the tangent line to the graph of $y = x^2$ at $(2,4)$.

<u>Solution</u> The secant line from $(2,4)$ to $(x, f(x)) = (x, x^2)$ has slope

$$\frac{\Delta y}{\Delta x} = \frac{x^2 - 4}{x-2} = \frac{(x+2)(x-2)}{x-2} = x+2 \quad \text{for } x \neq 2.$$

The slope of the tangent line is

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} (x+2) = 2+2 = 4.$$